

Entrepreneurship, Financial Frictions, and the Market for Firms

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Motivation

- Financial frictions: entrepreneurs finance new projects using own resources
- Who are the owners of private firms matter for allocations

⇒ **Entrepreneurs might want to sell their firms to less constrained parties**

- **Questions**
 - ▶ Is there evidence of this type of trades in the data?
 - ▶ How important are these trades for the aggregate economy?

This Paper

- Document relevant features of the trade of privately held firms in the US
- Develop a GE **model** of entrepreneurship and frictional trade of firms
- Validate our theory in the data: **financial frictions** + **motive to trade firms**
- Use the model to quantify how important is the trade of firms and study how credit conditions can affect this market

Related Literature

- **Entrepreneurship and the wealth distribution**
 - ▶ Cagetti and De Nardi (2006), Peter (2019)
- **Finance and misallocation**
 - ▶ Buera, Kaboski and Shin (2011), Midrigan and Xu (2014)
- **Market of ideas/patents**
 - ▶ Silveira and Wright (2010), Akcigit, Celik and Greenwood (2016)

Outline

Some Facts About the Trade of Firms

A Model of Entrepreneurship and Trade of Firms

Workings of the Model

Parameterization and Validation

Quantitative Analysis

Final Remarks

Data Sources

- Survey of Business Owners PUMS (**SBO**) [2007]
 - ▶ Information on how owners acquired their firm and firm characteristics
- Survey of Consumer Finances (**SCF**) [1989 : 3 : 2016]
 - ▶ Time series on how firm owners acquired their firm, and moments for income and wealth
- Annual Survey of Entrepreneurs (**ASE**) [2014 : 2016]
 - ▶ Complementary data on how owners acquired their firm
- Kauffman Firm Survey (**KFS**) Panel-[2004 : 2011]
 - ▶ Information about firms' balance sheet before trade

How do Entrepreneurs Acquire Their Firms?

- In 2007, **1/5** of entrepreneurs acquired their firm by purchasing an existing business

Share of entrepreneurs, by type of acquisition

		Founded	Purchased	Inherited/Other
Entrepreneur	SBO	77.0%	17.0%	6.2%
	SCF	71.9%	17.7%	10.5%
+ Employment > 0	SBO	65.2%	25.5%	9.7%
	SCF	65.3%	22.7%	12.0%

NOTES: Entrepreneurs are defined as (1) self-employed, (2) business owners, who (3) actively manage their firm.

SOURCE: 2007 Survey of Business Owners (SBO) and 2007 Survey of Consumer Finances (SCF).

► BizBuySell

► Franchises

► Robustness

► Sectors

- Annual trade rate of **2-3%**

Previous Occupation of Firms' Buyers

The **SBO** provides information about entrepreneurs' previous occupation

- Between **62-66%** of buyers were **employees before purchasing** the firm
- Buying an existing firm is a relevant channel for entering into entrepreneurship

Trade of Firms Across Time

Fraction of entrepreneurs that purchased their business

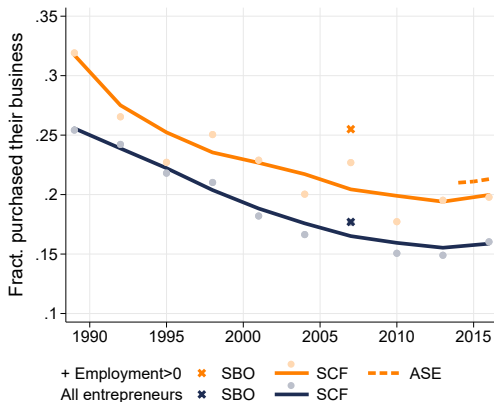


SOURCE: Survey of Business Owners (SBO), Survey of Consumer Finances (SCF) and Annual Survey of Entrepreneurs (ASE).

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Environment

Time

- Discrete and infinite
- Each period is divided into two: the market for firms and the production stage

Commodity space and financial markets

- Final consumption good c
- Risk free asset a , for savings and as a medium of exchange in the market for firms
- Incomplete markets (uninsurable idiosyncratic risk) and borrowing constraints

Agents and technology

- Measure of **households** in $[0, 1]$, preferences over consumption
- **Private firms**, owned by a single household, can be traded in the market for firms
- **Public firm** and a **financial intermediary**, both owned by all households in equal shares

Households' Endowments and Occupations

Firm owners

- Are endowed with a **private firm of quality** z , which evolves according to

$$z' = \begin{cases} z & \text{w/ pr } \gamma \\ z' \sim \mathcal{P}(z_{\min}, \eta_z) & \text{w/ pr } (1 - \gamma) \end{cases}$$

- Can produce the consumption good with technology $y = z f(k, l)$
- What's z ? Firm's intangible assets (trademarks, patents, processes, customer bases)

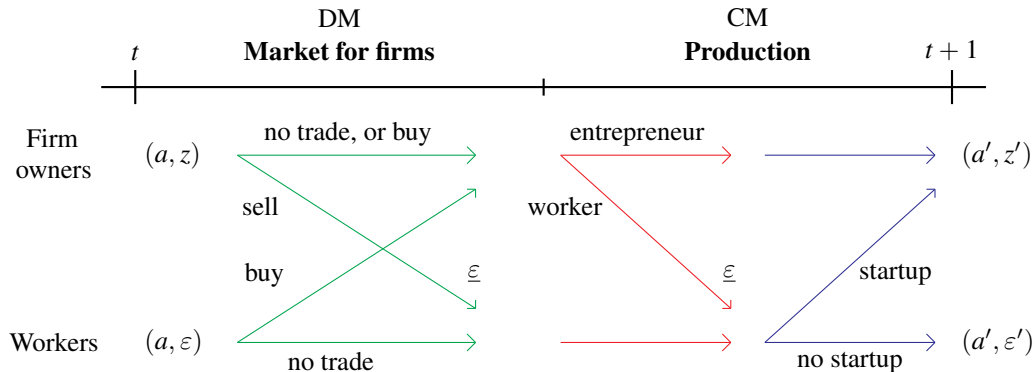
► BizBuySell Examples

Workers

- Are endowed with one unit of labor and a **labor efficiency** ε which follows

$$\log \varepsilon' = \rho_{\varepsilon} \log \varepsilon + \sigma_{\varepsilon} u, \quad u \sim \mathcal{N}(0, 1)$$

Transitions Between Occupations



Firms and the Financial Intermediary

- If a firm owner operates the profits of the **private firm** are

$$\pi(a, z) = \max_{k, l} z \left(k^\eta l^{(1-\eta)} \right)^\Upsilon - (r + \delta)k - wl$$
$$\text{s.t. } k \leq \lambda a$$

where $\Upsilon < 1$, and $\lambda \geq 1$ characterizes the **collateral constraint** on owner's assets a

- The representative **public firm** solves

$$\max_{K_c, L_c} \Pi_c = K_c^\eta L_c^{1-\eta} - (r + \delta)K_c - wL_c$$

- The **financial intermediary** takes deposits from HHs and rent capital to firms

► Details

A Market for Firms (1/2)

- Firms are hard to evaluate and price
- **Search-theoretic** approach
 - ▶ Bilateral random matching and *quid pro quo* trade
 - ▶ Intuition: potential buyers can evaluate only one firm per period
- Two types of meetings: **owner-owner** and **owner-worker**
 - ▶ Meeting probabilities conditional on occupation: α_o and α_w
 - ▶ Owner-owner meeting relative firm qualities determine who buys/sells
if $z < \tilde{z}$, (a, z) is the buyer and $(\tilde{a}, \tilde{z})$ is the seller

A Market for Firms (2/2)

- If total surplus > 0 , buyer and seller **Nash bargain** over the price p
- Let $\underline{p}$ be seller's minimum price (seller's surplus = 0) and $\bar{p}$ buyer's maximum price (buyer's surplus = 0), the **condition for trade** is

$$\underline{p}(\tilde{\mathbf{s}}) < p(\tilde{\mathbf{s}}, \mathbf{s}) < \bar{p}(\mathbf{s}, \tilde{\mathbf{z}})$$

where $\tilde{\mathbf{s}} \equiv (\tilde{a}, \tilde{z})$, $\mathbf{s} \in \{(a, z), (a, \varepsilon)\}$

- Assume **buyer has all the bargaining power** (seller's surplus = 0)

$$p(\tilde{\mathbf{s}}, \mathbf{s}) = \underline{p}(\tilde{\mathbf{s}})$$

► Nash Bargaining

► Trade Surpluses

Value in the Market for Firms (DM)

- For **firm owners**, the value at the beginning of DM is

$$\begin{aligned}
 V^o(a, z) &= \Pr^o [\text{no trade}] W^o(a, z) && \text{(no trade)} \\
 &+ \alpha_o s_{dm}^o \int_{z < \tilde{z}, \underline{p} < \bar{p}} W^o(a - p, \tilde{z}) dN_{dm}^o(\tilde{a}, \tilde{z}) && \text{(buy)} \\
 &+ \alpha_o s_{dm}^o \int_{z > \tilde{z}, \underline{p} < \bar{p}} W^w(a + p, \underline{\varepsilon}) dN_{dm}^o(\tilde{a}, \tilde{z}) && \text{(sell-owner)} \\
 &+ \alpha_w (1 - s_{dm}^o) \int_{\underline{p} < \bar{p}} W^w(a + p, \underline{\varepsilon}) dN_{dm}^w(\tilde{a}, \tilde{\varepsilon}) && \text{(sell-worker)}
 \end{aligned}$$

- For **workers**, the value at the beginning of DM is

$$\begin{aligned}
 V^w(a, \varepsilon) &= \Pr^w [\text{no trade}] W^w(a, \varepsilon) && \text{(no trade)} \\
 &+ \alpha_w s_{dm}^o \int_{\underline{p} < \bar{p}} W^o(a - p, \tilde{z}) dN_{dm}^o(\tilde{a}, \tilde{z}) && \text{(buy)}
 \end{aligned}$$

Value in the Production Stage (CM)

- The value of being a **firm owner** at the beginning of CM is

$$W^o(a, z) = \max_e \{W^e(a, z), W^w(a, \underline{\varepsilon})\}$$

where W^e is value the of being an **entrepreneur**

$$\begin{aligned} W^e(a, z) = \max_{a', c} & u(c) + \beta \{ \gamma V^o(a', z) + (1 - \gamma) \mathbb{E}_{z'} [V^o(a', z')] \} \\ \text{s.t. } & c = \pi(a, z) + (1 + r)a - a' \\ & c \geq 0, a' \geq 0 \end{aligned}$$

- The value of being a **worker** at CM is

$$\begin{aligned} W^w(a, \varepsilon) = \max_{a', c} & u(c) + \beta \{ \zeta \mathbb{E}_{\varepsilon' | \varepsilon} [V^w(a', \varepsilon')] + (1 - \zeta) \mathbb{E}_{z'} [V^o(a', z')] \} \\ \text{s.t. } & c = \varepsilon w + (1 + r)a - a' \\ & c \geq 0, a' \geq 0 \end{aligned}$$

Equilibrium

A **competitive equilibrium** consists of: *(i)* aggregate prices; *(ii)* terms of trade in the DM; *(iii)* occupational choice of firm owners; *(iv)* consumption and savings decisions for households; *(v)* capital and labor choices of firms; and *(vi)* measures of agents over types and idiosyncratic states at DM and CM such that:

1. In DM, the terms of trade in bilateral meetings are solved by the bargaining problem
2. In CM, given prices, households, private and corporate firms solve their optimization problems
3. Goods and labor market clears [▶ Detail](#)
4. The financial intermediary breaks even [▶ Detail](#)
5. The law of motion of n_{dm} and n_{cm} are consistent with the trades of firms, agents' optimal choices and the laws' of motion for the exogenous processes

[▶ Solution Method](#)

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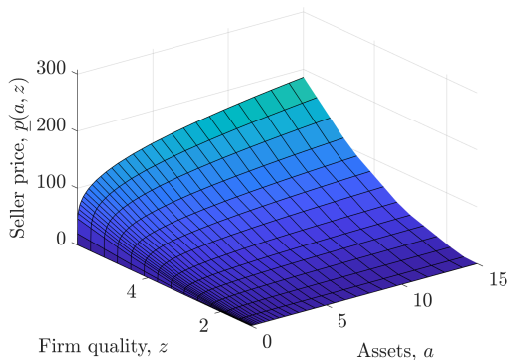
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Pricing of Private Firms

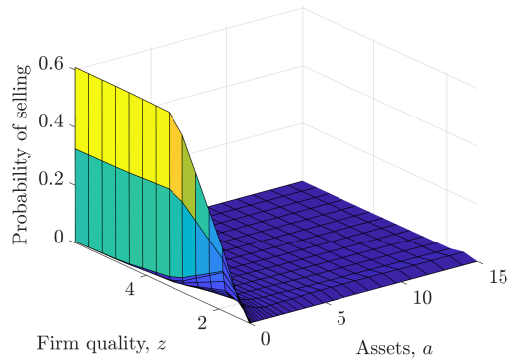
If the buyer has all the bargaining power $\underline{p} = \underline{p}$

Sellers' minimum price $\underline{p}(a, z)$



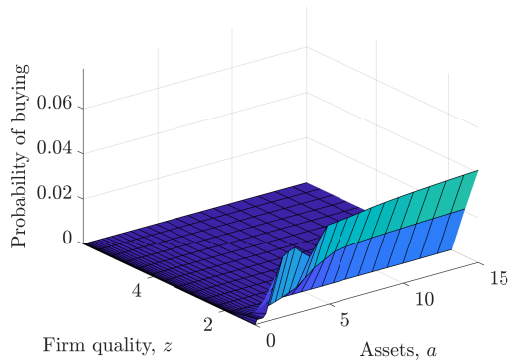
Who Sell Firms?

Probability of selling

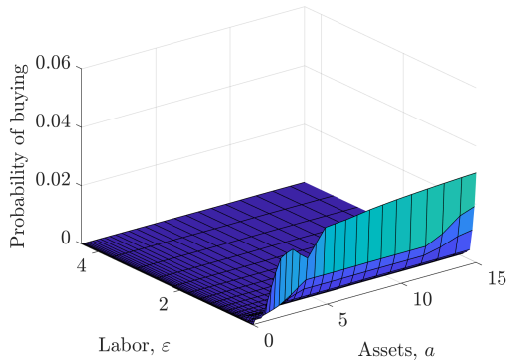


Who Buy Firms?

(a) Probability of buying, **owners**



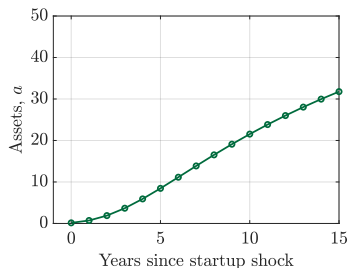
(b) Probability of buying, **workers**



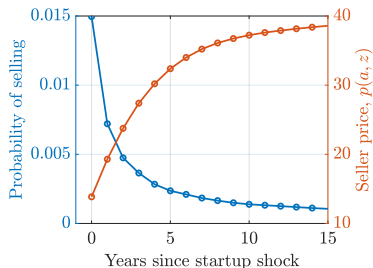
Firm Dynamics and Trade: An Example

- Initial owner assets = median worker in the economy
- Firm quality z is constant across t and = 3rd best firm in grid

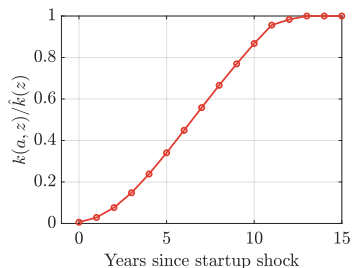
(a) Assets of current owner



(b) Selling probability and price



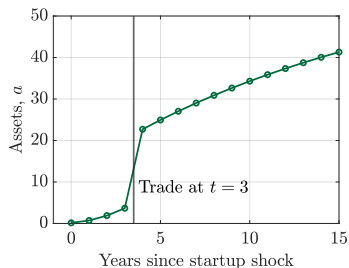
(c) k relative to unconstrained



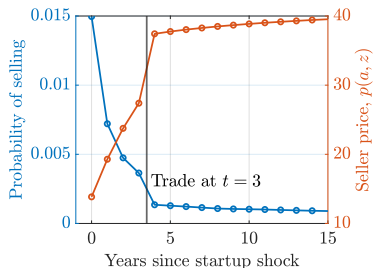
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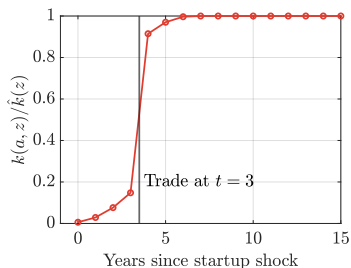
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Calibration Strategy

- Exogenously assign σ , δ and η to standard values, and set β such that $K_c/Y_c = 3$
- Remaining parameters are chosen to match several features of the US economy in **2007**

Assigned Parameters		
	Value	Description
σ	1.5	CRRA
δ	0.06	Capital depreciation rate
η	1/3	Capital elasticity

Calibrated Parameters		
	Value	Description
Υ	0.708	Curvature private firms technology
λ	2.001	Collateral constraint
γ	0.925	Persistence private firm value
ζ	0.933	1 – Startup shock
z_{min}	1.166	Scale, z distribution
η_z	2.827	Shape, z distribution
ρ_ε	0.964	AR(1) parameter, ε distribution
σ_ε	0.160	Std. Deviation, ε distribution
α_o	0.802	Owner-owner meeting probability
α_w	0.603	Owner-worker meeting probability

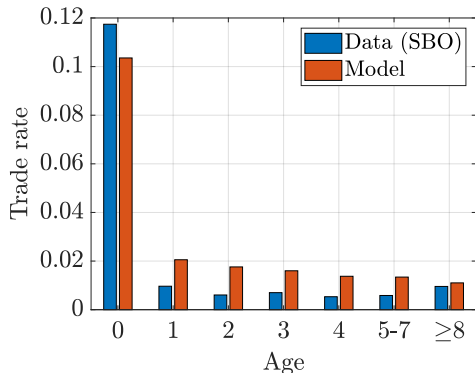
Targeted Moments

► Other Untargeted Moments

	Data	Model
Fraction of entrepreneurs	0.08	0.09
Debt to capital, weighted	0.35	0.33
Private firms output share	0.48	0.46
Entrepreneurship exit rate	0.08	0.08
Income share, entrepreneurs	0.21	0.21
Wealth share, entrepreneurs	0.35	0.32
Gini income, all households	0.57	0.47
Gini wealth, all households	0.79	0.84
Gini income, entrepreneurs	0.66	0.65
Gini wealth, entrepreneurs	0.75	0.77
Gini income, workers	0.53	0.41
Gini wealth, workers	0.76	0.84
Fraction of firms traded	0.18	0.19
Annual trade rate	0.02	0.02
Share of firms purchased by workers	0.62	0.62

Validation: Financial Frictions as a Motive for Trade

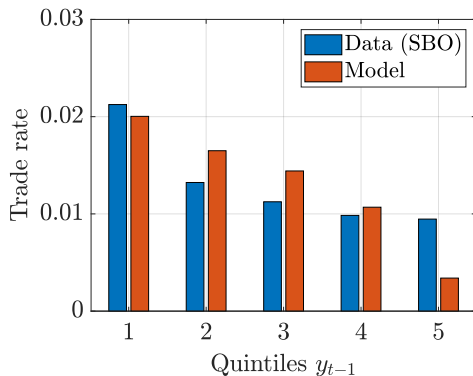
(a) Trade vs. firm age



NOTES: Data from the 2007 SBO. Trade is computed using the fraction of owners that acquired their firm through a **purchase** in 2007. The age of the firm is computed as the difference between 2007 and the year when the business was established.

Validation: Financial Frictions as a Motive for Trade

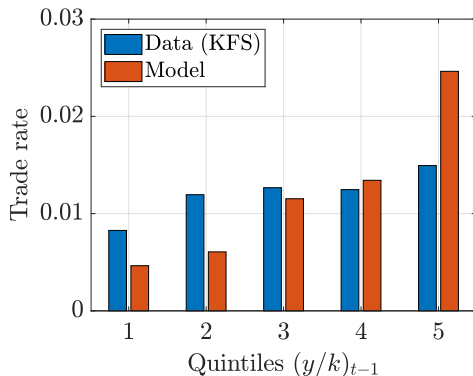
(b) Trade vs. size



NOTES: Data from the 2007 SBO. Trade is computed using information from all the firms that were **sold** in or after 2007. Firm size is measured by total sales in 2007.

Validation: Financial Frictions as a Motive for Trade

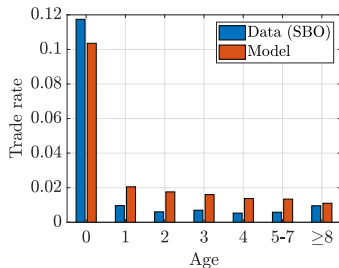
(c) Trade vs. APK



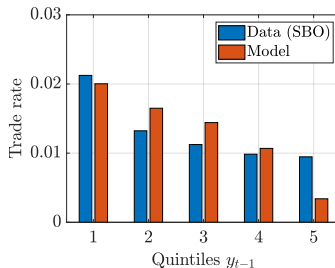
NOTES: Data from the public version of the KFS. Trade is computed using information from all the firms that were **sold** during the years of the sample. Average productivity of capital (APK) is measured by sales over capital of the year previous the sell. Capital includes inventories, equipment and machinery, land, buildings, and structures, vehicles and other assets owned by the business. The relation is computed for every year and then averaged across time.

Validation: Financial Frictions as a Motive for Trade

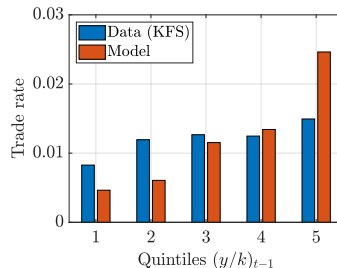
(a) Trade vs. firm age



(b) Trade vs. size



(c) Trade vs. APK



Additional evidence

► Size After Trade

► Trade vs. Seller's Age

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The Role of the Market of Firms: Closing the Market

	Baseline	$(\alpha_o, \alpha_w)/2$	$(\alpha_o, \alpha_w) = \mathbf{0}$
Fract. firms traded	0.19		
Fract. firms purchased by workers	0.64		
Fract. entrepreneurs	0.09		
Δ Interest rate			
Δ Wages			
Δ Output			
Δ Output, public			
Δ Output, entrepreneurial			
Δ TFP, entrepreneurial			
Wealth top 1	0.32		
Entrepreneurs' wealth share	0.32		
Wealth top 1, entrepreneurs	0.26		

The Role of the Market of Firms: Closing the Market

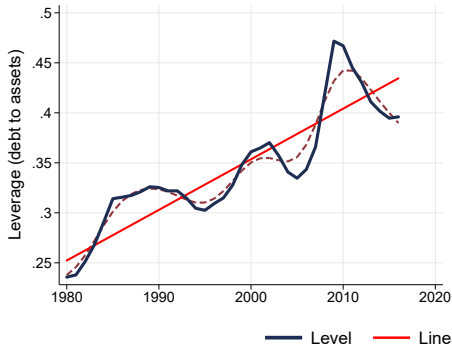
	Baseline	$(\alpha_o, \alpha_w)/2$	$(\alpha_o, \alpha_w) = \mathbf{0}$
Fract. firms traded	0.19	0.11	
Fract. firms purchased by workers	0.64	0.65	
Fract. entrepreneurs	0.09	0.08	
Δ Interest rate		0.58%	
Δ Wages		-0.13%	
Δ Output		-0.1%	
Δ Output, public		2.0%	
Δ Output, entrepreneurial		-2.5%	
Δ TFP, entrepreneurial		-0.6%	
Wealth top 1	0.32	0.32	
Entrepreneurs' wealth share	0.32	0.29	
Wealth top 1, entrepreneurs	0.26	0.28	

The Role of the Market of Firms: Closing the Market

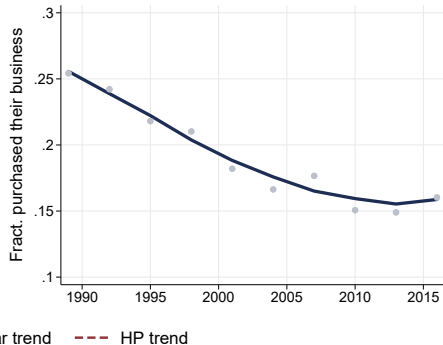
	Baseline	$(\alpha_o, \alpha_w)/2$	$(\alpha_o, \alpha_w) = \mathbf{0}$
Fract. firms traded	0.19	0.11	0.00
Fract. firms purchased by workers	0.64	0.65	-
Fract. entrepreneurs	0.09	0.08	0.08
Δ Interest rate		0.58%	1.33%
Δ Wages		-0.13%	-0.31%
Δ Output		-0.1%	-0.2%
Δ Output, public		2.0%	4.9%
Δ Output, entrepreneurial		-2.5%	-5.9%
Δ TFP, entrepreneurial		-0.6%	-1.5%
Wealth top 1	0.32	0.32	0.32
Entrepreneurs' wealth share	0.32	0.29	0.25
Wealth top 1, entrepreneurs	0.26	0.28	0.31

Credit Conditions and the Trade of Firms (1/2)

(a) Noncorporate firms leverage



(b) Fract. of entrepreneurs that purchased

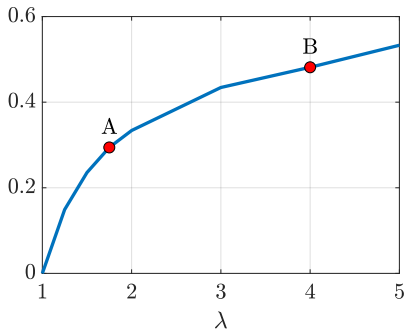


SOURCE: Flow of Funds Accounts and Survey of Consumer Finances (SCF).

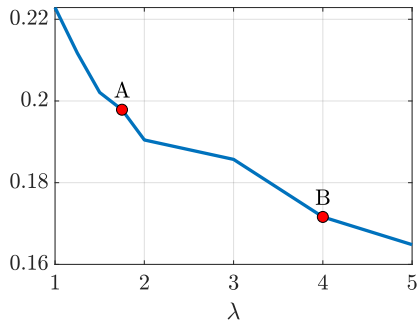
Credit Conditions and the Trade of Firms (2/2)

- Given the change in debt to capital (~ 20 p.p.), our model suggests that **3 out of 10 p.p.** drop in the share of traded firms can be explained by easier access to credit

(a) Debt to capital, weighted



(b) Fraction of firms purchased



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Summary

- We documented that **1/5** of entrepreneurs **purchased** their firm and around **60%** of them were **workers** before purchasing
- We showed, in theory and data, that **financial frictions** are +motive to trade firms
- The results from our quantitative model shows that **the trade of firms is a relevant mechanism** through which entrepreneurial projects and available resources are **allocated**
 - ▶ Shutting down this market implies an entrepreneurial output fall of around 6%
- Looser credit conditions can explain for 3 out of the 10 p.p. of the fall in the share of traded firms observed during the last 30 years

Next Steps - Research Agenda

Implications for fiscal policy

- Our setup is suitable to study the implications of wealth taxes on businesses, or taxes to the trade of firms (capital gains taxes)

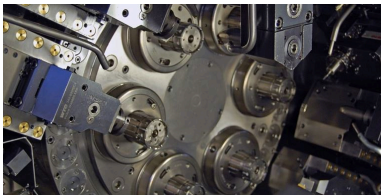
Evidence on firm dynamics and the trade of firms using Census data

- Our model has various implications for firm dynamics after a trade
- We want to test these implications using data from the two waves of the **SBO** (2007 and 2012) and the panel of firms in the Longitudinal Business Database (**LBD**)

Thanks!

Businesses For Sale: Some Examples

Multi-State Wholesaler of Industrial Machinery, Equipment

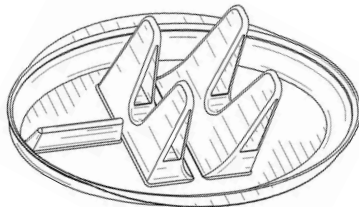


Asking Price: **\$8,900,000**

Support & Training: The owners will provide extensive training and transition for several years in a step down to retirement plan.

Reason for Selling: Growth calls for support from outside larger ownership investment

The Hot Dog/Pita plate



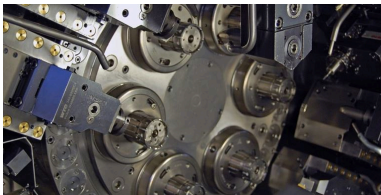
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United States Design Patent

Patent No.: **US D793,163 S**

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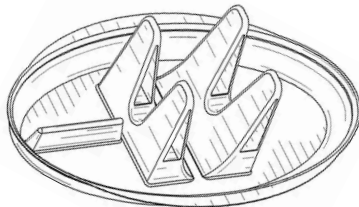


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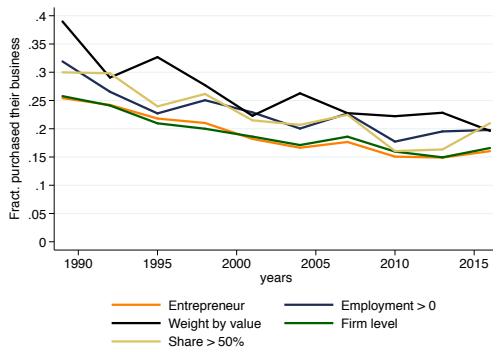
Patent No.: **US D793,163 S**

Firm Acquisition, Alternative Definitions and Samples

<i>Moment</i>	<i>% purchased</i>	<i>N (owners)</i>	<i>min</i>	<i>max</i>	<i>N (firms)</i>
respond acquisition	16.0%	20,302,192	13.4%	14.7%	13,793,882
manage	17.0%	9,503,681	15.7%	16.3%	7,723,096
>0 employment	25.9%	5,507,460	23.9%	26.8%	3,281,041
>0 receipts	16.9%	17,139,950	14.2%	15.7%	11,445,027
>0 all size	26.1%	5,344,965	24.0%	27.1%	3,176,929
manage + employment > 0	25.5%	3,167,718	24.5%	25.7%	2,424,327
manage + payroll > 0	24.7%	3,473,610	23.6%	24.8%	2,676,999
share ≥ 50 + employment > 0	23.5%	3,884,071	22.7%	23.3%	3,009,027
share ≥ 50 + manage	15.4%	8,064,388	14.7%	14.9%	7,061,037
share ≥ 50 + size > 0 + manage	24.2%	2,385,664	23.6%	24.0%	2,047,708
baseline + wgt by payroll	32.4%	3,167,718	30.4%	-	2,424,327
operating	25.6%	3,167,718	24.6%	25.9%	2,424,327
not operating	21.6%	3,167,718	21.3%	22.0%	2,424,327

Trade of Firms Across Time, Alternative Definitions and Samples

Fraction of entrepreneurs that purchased their business



SOURCE: Survey of Consumer Finances (SCF).

NOTES: Entrepreneurs are defined as (1) self-employed, (2) business owners, who (3) actively manage their firm.

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Firm Size After Purchased

Firm Size and Purchased, *Kitchen Sink* Regressions

$$\ln(\text{receipts}) = X\beta + \varepsilon$$

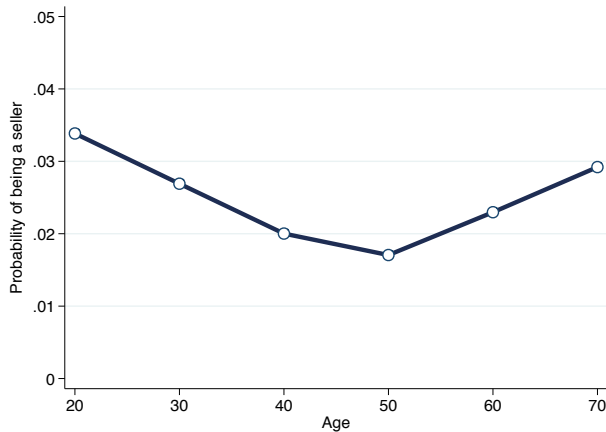
	(1)	(2)
Purchased	13.78*** (3.250)	20.85*** (1.845)
Age firm x Purchased	-0.0766 (0.106)	-0.353** (0.138)
Age firm	2.834*** (0.0832)	2.322*** (0.102)
Observations	443668	433680
R^2	0.050	0.160
Controls	No	Yes

SOURCE: SBO-PUMS.

NOTES: Sample is restricted to entrepreneurs of businesses that have a positive payroll, employment and receipts. Standard errors clustered by state and sector are presented in parentheses.

Sellers' Age

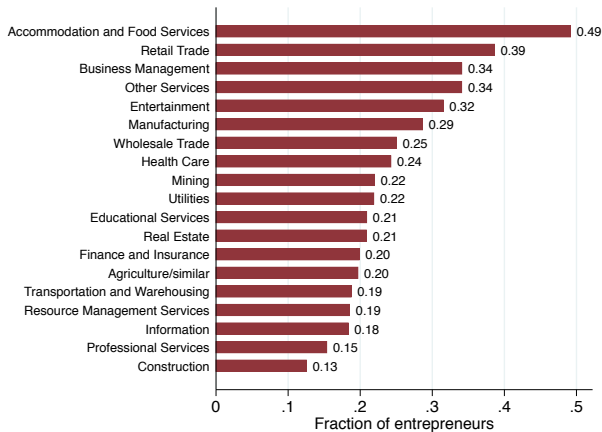
Fraction of firm by sellers' age (SBO)



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Firm Trade, by Sector

Fraction of entrepreneurs that purchased



Franchises

Percentage of entrepreneurs that purchased their firm

	Entrepreneur	+ Employment > 0
Baseline	17.0%	25.5%
W/o franchises	16.1%	24.1%
Franchises only	50.1%	51.8%
Franchises % of total	2.8%	4.8%

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Market Clearing

- Goods' market clearing (feasibility):

$$Y = C + K' - (1 - \delta) K$$

where

$$Y \equiv Y_c + s_{cm}^e \int z \left(k(a, z)^\eta l(a, z)^{(1-\eta)} \right)^\Upsilon dN_{cm}^e(a, z)$$

$$K \equiv K_c + s_{cm}^e \int k(a, z) dN_{cm}^e(a, z)$$

$$C \equiv s_{cm}^e \int c(a, z) dN_{cm}^e(a, z) + s_{cm}^w \int c(a, \varepsilon) dN_{cm}^w(a, \varepsilon)$$

- Labor market clearing:

$$L_c + s_{cm}^e \int l(a, z) dN_{cm}^e(a, z) = s_{cm}^w \int \varepsilon dN_{cm}^w(a, \varepsilon)$$

Financial Intermediary

- The intermediary takes deposits from households and rent capital to the firms at a price equal to the savings rate plus capital depreciation: $r + \delta$
- It operates in a perfectly competitive market and breaks even (zero profits)
- The resource constraint of the intermediary is given by

$$K_c + s_{cm}^e \int k(a, z) \, dN_{cm}^e(a, z) = s_{cm}^e \int a \, dN_{cm}^e(a, z) + s_{cm}^w \int a \, dN_{cm}^w(a, \varepsilon)$$

Trade Surpluses

Owner-owner

- If $z < \tilde{z}$, $\mathbf{s}^o \equiv (a, z)$ is the buyer and $\tilde{\mathbf{s}} \equiv (\tilde{a}, \tilde{z})$ is the seller

$$\text{Total surplus} \equiv \underbrace{W^o(a - p, \tilde{z}) - W^o(\mathbf{s}^o)}_{\text{Buyer's surplus}} + \underbrace{W^w(\tilde{a} + p, \underline{\varepsilon}) - W^o(\tilde{\mathbf{s}})}_{\text{Seller's surplus}}$$

Owner-worker

- The worker $\mathbf{s}^w \equiv (a, \varepsilon)$ is the buyer and the firm owner $\tilde{\mathbf{s}} \equiv (\tilde{a}, \tilde{z})$ is the seller

$$\text{Total surplus} \equiv \underbrace{W^o(a - p, \tilde{z}) - W^w(\mathbf{s}^w)}_{\text{Buyer's surplus}} + \underbrace{W^w(\tilde{a} + p, \underline{\varepsilon}) - W^o(\tilde{\mathbf{s}})}_{\text{Seller's surplus}}$$

Nash Bargaining Problem

- If total surplus > 0 , p solves

$$p(\tilde{\mathbf{s}}, \mathbf{s}) = \arg \max_p \left[S_b(\mathbf{s}, \tilde{z}, p) \right]^\chi \left[S_s(\tilde{\mathbf{s}}, p) \right]^{1-\chi}$$
$$\text{s.t. } S_b(\mathbf{s}, \tilde{z}, p) \geq 0, S_s(\tilde{\mathbf{s}}, p) \geq 0$$

where $\tilde{\mathbf{s}} \equiv (\tilde{a}, \tilde{z})$, $\mathbf{s} \in \{(a, z), (a, \varepsilon)\}$, S_b and S_s are buyer's and seller's surpluses

- If $\chi = 1$ **buyer has all the bargaining power**

Solution Method

- We solve the model using projection methods

$$V^z(a, z) = \Phi^z(a, z)g_V^z$$

$$W^z(a, z) = \Phi^z(a, z)g_W^z$$

$$V^w(a, \varepsilon) = \Phi^\varepsilon(a, \varepsilon)g_V^w$$

$$W^w(a, \varepsilon) = \Phi^\varepsilon(a, \varepsilon)g_W^w$$

- FOC of the public firm implies a relation b/t Y_c/K_c , w and r , thus only need to solve for one price: r
- Then, the equilibrium objects we need to solve for are

$$\{g_V^z, g_W^z, g_V^w, g_W^w, n_{dm}^z, n_{dm}^w, n_{cm}^z, n_{cm}^w, P_{dm}^z, P_{dm}^w, P_{cm}^z, P_{cm}^w, r\}$$

where n and P denotes the density and transition probability matrix across states

Algorithm

Iteration on prices

0. Propose an initial guess for r .
1. Given r , solve the model (in partial equilibrium).

Iteration on distributions

- 1.0. Propose an initial guess for $\{n_{dm}^z, n_{dm}^w\}$.
- 1.1. Given $\{n_{dm}^z, n_{dm}^w\}$, solve for $\{g_W^z, g_W^w\}$.

Iteration on value functions

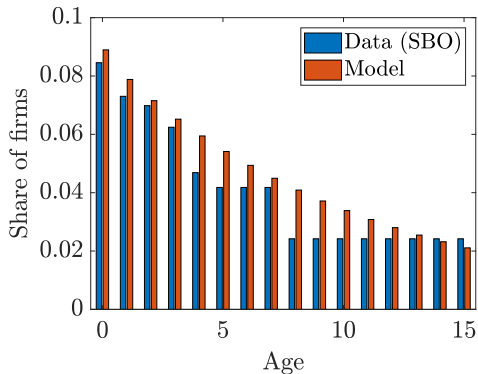
- 1.1.0. Propose an initial guess for $\{g_W^z, g_W^w\}$.
- 1.1.1. Solve the DM problem: get $\{g_V^z, g_V^w\}$.
- 1.1.2. Solve the CM problem: obtain e, a' and P_{cm} .
- 1.1.3. Update $\{g_W^z, g_W^w\}$.
- 1.1.4. Iterate $\{g_W^z, g_W^w\}$ until convergence.
- 1.2. Update $\{n_{dm}^z, n_{dm}^w\}$.
- 1.3. Iterate $\{n_{dm}^z, n_{dm}^w\}$ until convergence.
2. Update r using bisection on the labor market clearing condition.
3. Iterate r until the labor market clears.

Other Untargeted Moments

Wealth and Income Distribution

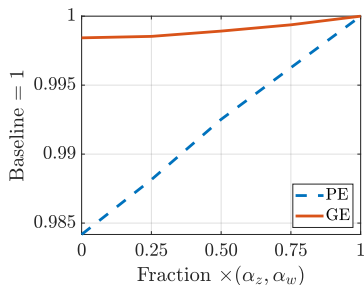
	Data	Model
Wealth, all households		
Top 1	0.33	0.32
Top 10	0.72	0.73
Bottom 50	0.02	0.01
Bottom 25	0.00	0.00
Income, all households		
Top 1	0.21	0.16
Top 10	0.47	0.38
Bottom 50	0.14	0.18
Bottom 25	0.04	0.08

Firms age distribution

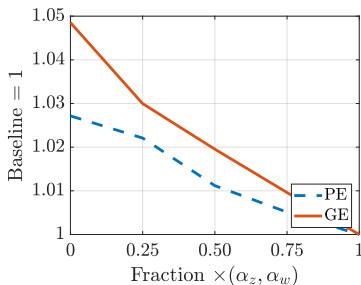


Closing the Market: GE vs PE

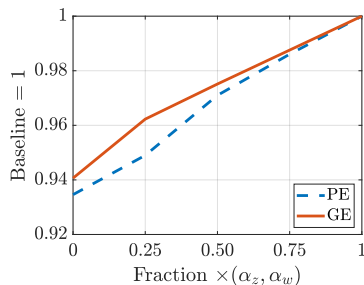
(a) Total output



(b) Corporate output



(c) Entrepreneurial output

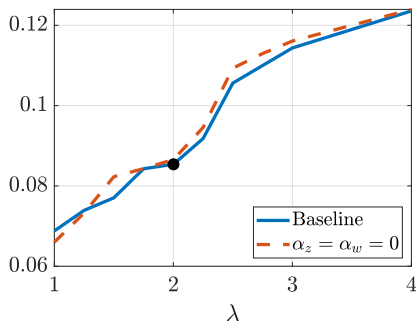


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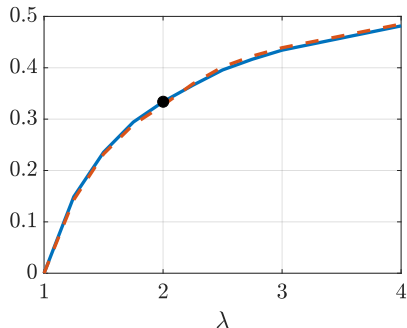
The Role of the Market of Firms: Baseline vs No Market (1/2)

- Compare our baseline model vs an economy with no market for firms
 - ▶ Calibrate the no market economy to the same moments used for the baseline
- Solve the model for different degrees of credit conditions ($\Delta\lambda$) and do ss comparisons

(a) Fraction of entrepreneurs



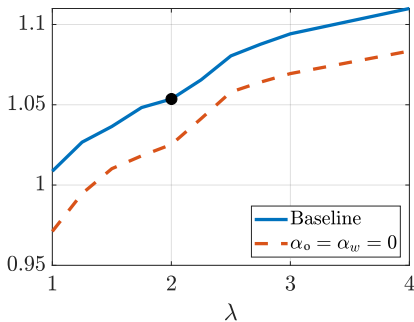
(b) Debt to capital, weighted



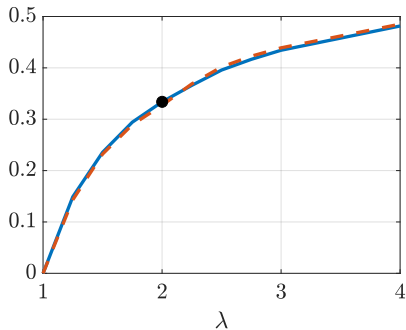
The Role of the Market of Firms: Baseline vs No Market (2/2)

- What's the credit level the no market economy requires in order to match the entrepreneurial **TFP** in the baseline economy?

(a) TFP, entrepreneurial

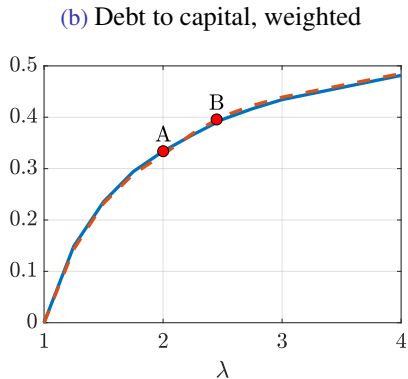
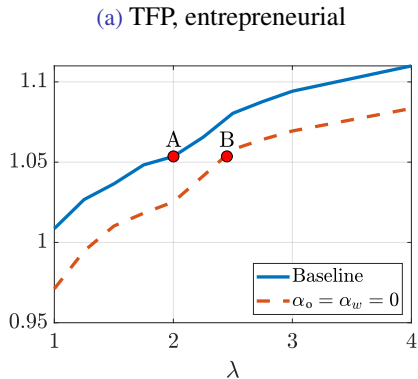


(b) Debt to capital, weighted



The Role of the Market of Firms: Baseline vs No Market (2/2)

- What's the credit level the no market economy requires in order to match the entrepreneurial **TFP** in the baseline economy?



- Need **looser credit** conditions such that firms' debt to capital is ~ 8 p.p. higher!